Topics 13c + 16 vs. Topic 22a & b

One Population

$\mu = ?$ $\sigma = ?$

Looking at population mean μ

Sample

$n = 34$ $\bar{x} = 56.7$ $s = 5.8$

Two Populations

$\mu_1 = ?$ $\sigma_1 = ?$ $\mu_2 = ?$ $\sigma_2 = ?$

Looking at the difference in the population means $\mu_1 - \mu_2$

TAKE Samples

Sample One: $n_1 = 43$ $\bar{x}_1 = 82.5$ $s_1 = 9.6$

Sample Two: $n_2 = 37$ $\bar{x}_2 = 79.2$ $s_2 = 10.7$

Best Point Estimate: $\bar{x}$ vs. $\bar{x}_1 - \bar{x}_2$

Distribution of the POINT ESTIMATES

Student's-t with $n-1$ degrees of freedom

$\frac{s}{\sqrt{n}}$

Standard deviation of the POINT ESTIMATE

Student's-t with ? degrees of freedom

$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Confidence Interval

$\bar{x} \pm \left(t_{\frac{\alpha}{2}, df}\right)\left(\frac{s}{\sqrt{n}}\right)$

$56.7 \pm t_{\frac{\alpha}{2}, 33}\left(\frac{5.8}{\sqrt{34}}\right)$

90%

5% below, 5% above

qt(.05, 33, lower.tail = FALSE) gives the value 1.692

$56.7 \pm 1.692\left(\frac{5.8}{\sqrt{34}}\right)$

$(55.017, 58.383)$

$(\bar{x}_1 - \bar{x}_2) \pm \left(t_{\frac{\alpha}{2}, df}\right)\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

We need to choose a value for the degrees of freedom!

Simple Choice: One less than the smaller of n_1 and n_2

Simple Continued

degrees of freedom set to 36

for 90% C.I.
qt(0.05, 36, lower.tail = FALSE)

gives 1.688

$$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2},36} \sqrt{\frac{9.6^2}{43} + \frac{10.7^2}{37}}$$

$$(82.5 - 79.2) \pm 1.688 * 2.2886$$

$$(-0.563, 7.163)$$

Complex a Computed Value for Degrees of Freedom

find $d_1 = \frac{s_1^2}{n_1}$ and $d_2 = \frac{s_2^2}{n_2}$

$$d_1 = \frac{9.6^2}{43} \qquad d_2 = \frac{10.7^2}{37}$$

$$d_1 = 2.143256 \qquad d_2 = 3.094324$$

Compute

$$df = \frac{(d_1 + d_2)^2}{\frac{d_1^2}{n_1 - 1} + \frac{d_2^2}{n_2 - 1}} = 73.086$$

use 73 degrees of freedom
qt(0.05, 73, lower.tail = FALSE)
gives 1.666

The 90% confidence INTERVAL

$$(82.5 - 79.2) \pm 1.666 * 2.2886$$

$$(-0.513, 7.113)$$